

Transport Risk Models for Hazardous Materials: Revisited

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Abstract:

We study several functions that have been proposed for comparing transport routes for a hazardous materials shipment. We show that many proposed path evaluation functions are based on an approximation. While these functions can be modified to avoid the approximation, these more accurate versions violate two reasonable axioms. We propose a new family of models by redefining the decision problem as one of satisfying demand for a hazardous material at the destination point. The new models satisfy the axioms.

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Introduction

Transport planning for hazardous materials (hazmat) is an increasingly popular topic in operations research (Erkut and Verter, 1995). While published transport planning models focus on a variety of different issues, many have one component in common: a function that quantifies the risk imposed by a hazmat shipment on people living near the transport route. Different models quantify risk in different ways and there does not seem to be any consensus on how to do this (Erkut and Verter, 1995). Some proposed risk functions have been shown to behave in undesirable ways on pathological examples (Erkut, 1995). In this article, we take a critical look at how hazmat transport risk is quantified. We begin by reviewing the most common risk models in the literature. Then we consider two axioms and check which models satisfy these axioms. Consideration of the ways in which some of the models fail to satisfy the axioms leads us to propose a new set of risk models.

Models of Hazmat Transport Risk

The risk of transporting a hazardous material is usually quantified with a path evaluation function. Consider a path r consisting of an ordered set of links $\{1, 2, \dots, n\}$ and assume that each link has two important and known attributes: p_i , the probability of a release accident on link i , and C_i , a measure of the consequence of a release accident on link i . The consequence could, for example, be quantified as the number of people living within a mile of an accident site. The most common path evaluation function, which we term the “traditional risk model,” is:

$$\text{TR}(r) = \sum_{i=1}^n p_i C_i \quad (1)$$

The expression for traditional risk can be interpreted to be the expected value of the consequence of a hazmat truck traveling along path r . Use of this expression makes the tacit assumption that the truck will travel along every link on the path, regardless of what happened on earlier links. This is at odds with the reality that a release accident will often terminate the trip. To incorporate trip termination, we can replace the probability p_i of an accident on link i , *given* that the truck travels

along link i , with the expression $(1 - p_1)(1 - p_2) \cdots (1 - p_{i-1})p_i$, which includes the probability that the truck travels along links 1 to $i-1$ without accident. This leads to the following more complicated path evaluation function:

$$\text{TR}'(r) = \sum_{i=1}^n \prod_{j=1}^{i-1} (1 - p_j) p_i C_i \quad (2)$$

This function relaxes the assumption that a truck will travel along the entire path, but it continues to assume that if a truck enters a link, it will travel along the entire length of that link. If accidents along link i with length l_i , when traversed by a hazmat truck, are assumed to occur according to a spatial Poisson process with rate λ_i per distance unit, and if $p_i = l_i \lambda_i$, then we can account for the possibility of trip termination anywhere along a link as follows (Erkut and Ingolfsson, 2000):

$$\text{TR}''(r) = \sum_{i=1}^n \prod_{j=1}^{i-1} \exp(-p_j) (1 - \exp(-p_i)) C_i \quad (3)$$

If one defines link attributes $q_i = 1 - \exp(-p_i)$ for all links, then (3) reduces to the same form as (2), with q_i replacing p_i . Consequently, (2) and (3) have similar mathematical properties. The quantity q_i is the probability of one or more accidents occurring on link i when traversed by a hazmat shipment.

Expression (1) is an additive path evaluation function that leads to tractable shortest path problem formulations while (2) and (3) lead to difficult nonlinear binary integer programming formulations. Recent North American data on hazmat transport accidents indicate that such accident probabilities are extremely small, which causes the probability $1 - p_i$ that no accident occurs on link i to be close to 1, and this implies that (1) provides an excellent approximation to (2) and (3). While some authors start with (2) and approximate it with (1), others have used (1) without mentioning the inherent approximation.

Harwood, Viner, and Russel (1993), using North American data, estimate hazmat accident probabilities to be on the order of 10^{-6} per trip per kilometer. Using these estimates, Erkut and Verter (1998) estimate the error resulting from using (1) to approximate (2) to be about 0.125% for

a 2,500-mile path and conclude that the inaccuracy is negligible. Erkut and [Ingolfsson \(2000\)](#) provide an upper bound of $\exp(np_{\max}) - 1$ (where p_{\max} is an upper bound on the accident probability on any edge) on the percent error resulting from approximating (3) with (1) that leads to similar conclusions.

Extremely low accident probabilities result in small errors associated with using (1) instead of (2) or (3), but nevertheless we believe that further attention to hazmat risk modeling is justified. Although average accident probabilities on North American roads are very low, the same may not be true in other parts of the world depending on the quality of the infrastructure, driving habits, and regulations. Furthermore accident probabilities are higher at night ([Harwood et al., 1993](#)) or under adverse weather conditions, and an approximation that relies on low accident probabilities may result in poor path selection under such circumstances. In sum, the quality of the approximation depends on the magnitude of the accident probability data, and there is no good theoretical reason for using (1) instead of (2) or (3). This motivated our study of the fundamental properties of path evaluation functions proposed for hazmat transport risk and led us to discover interesting differences between expression (1) and expressions (2) and (3), which we describe in the next section.

Many path evaluation functions have been proposed in the literature in addition to the traditional risk model (see Erkut and Verter, 1998, for a survey). Table 1 summarizes eight path evaluation functions and cites representative studies that have used each model. In the table, D_i is used to denote the total population in the impact region along link i . One can think of D_i as the population in a rectangle that stretches along link i , in contrast to C_i , which one can think of as the number of people within a circle centered at any point on link i .

Traditional risk	$TR(r) = \sum_{i \in r} p_i C_i$	Alp (1995)
Population exposure	$PE(r) = \sum_{i \in r} D_i$	ReVelle et al. (1991)
Incident probability	$IP(r) = \sum_{i \in r} p_i$	Saccomanno and Chan (1985)
Perceived risk	$PR(r) = \sum_{i \in r} p_i C_i^q$	Abkowitz et al. (1992)
Mean-Variance	$MV(r) = \sum_{i \in r} (p_i C_i + k p_i C_i^2)$	Erkut and Ingolfsson (2000)
Disutility	$DU(r) = \sum_{i \in r} p_i (\exp(k C_i) - 1)$	Erkut and Ingolfsson (2000)
Minimax	$MM(r) = \max_{i \in r} C_i$	Erkut and Ingolfsson (2000)
Conditional risk	$CR(r) = \sum_{i \in r} p_i C_i / \sum_{i \in r} p_i$	Sivakumar et al. (1993a, 1993b)

Table 1: Eight path evaluation functions for hazmat transport risk that have appeared in the literature.

Each of the six models listed in Table 1 that use probabilities are based on the approximation inherent in (1). It is straightforward to develop alternate expressions along the lines of (2) and (3) for each of these models. We will refer to these alternate expressions as exact. Use of the approximate expressions in Table 1 for $TR(r)$, $IP(r)$, and $PR(r)$, as well as $PE(r)$ (which involves no probabilities), leads to shortest path problems with additive link attributes. Erkut and [Ingolfsson \(2000\)](#) show that $MV(r)$ and $DU(r)$ also reduce to shortest path problems and demonstrate that $MM(r)$ can be minimized using a variety of algorithms of similar complexity as shortest path algorithms. In contrast, $CR(r)$ is more difficult to optimize (see [Sivakumar et al., 1993b](#)).

As Table 1 illustrates, there are many ways to quantify hazmat transport risk, and it is not clear which model to choose. In the next section, we consider two axioms from [Erkut and Verter \(1998\)](#) in an effort to clarify the merit of the different models.

Hazmat Transport Risk Axioms

The two axioms we consider are relatively weak, and it seems desirable for hazmat path selection models to satisfy them. However, it turns out that some of the expressions in Table 1 violate one axiom or both.

We will use v to denote a non-negative path evaluation function. The two axioms prescribe properties for such functions.

Axiom 1: Path evaluation monotonicity: If path r' is contained in path r , then $v(r') \leq v(r)$.

This axiom implies that when links are added to a path its impact (value) cannot decrease. Commonly used path evaluation functions such as distance, time, and cost (when non-negative) satisfy this axiom, and it seems a reasonable requirement for hazmat transport risk path evaluation functions.

For Axiom 2, we assume v is expressed as a function of (possibly multiple) link attributes, i.e., $v(r) = f(u_1(r), u_2(r), \dots, u_k(r))$ where $u_i(r)$ are vectors of the same dimension as the number of links in the path r .

Axiom 2: Attribute monotonicity: If $h_i \geq 0$ for $i = 1, 2, \dots, k$, then

$$f(u_1(r), u_2(r), \dots, u_k(r)) \leq f(u_1(r) + h_1, u_2(r) + h_2, \dots, u_k(r) + h_k).$$

Axiom 2 states that if the value of any attribute for a particular link increases, all else being equal, the value of the path cannot decrease. In more concrete terms, if $u_1(r)$ is a vector of accident probabilities, $u_2(r)$ is a vector of consequences, and k equals 2, then Axiom 2 requires path risk to be a nondecreasing function of edge accident probabilities and consequences.

We now turn to the question of whether the risk models in Table 1 satisfy the axioms. [Erkut \(1995\)](#) and [Erkut and Verter \(1998\)](#) demonstrate that the conditional risk model violates both axioms. Of the remaining models in Table 1, six are additive path evaluation functions that use non-negative link attributes (traditional risk, population exposure, incident probability, perceived risk, mean-variance, and disutility) and satisfy both axioms. One can also verify that the minimax path evaluation function satisfies the axioms. However, recall that the models that involve probabilities rely on an approximation. Consider using the exact versions of the models that involve probabilities. Then some of these models violate the axioms. For example, the exact traditional risk model (2) violates both axioms, as illustrated in Figures 1 and 2. (In all figures, the accident probability for a link is shown above the link, and the consequence below the link.)

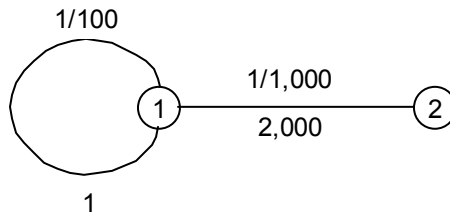


Figure 1: Example demonstrating that model (2) violates Axiom 1.

In Figure 1, the risk of travel along the link (1, 2) is $(0.001)(2000) = 2$, whereas the risk of travel along the link (1, 1), followed by the link (1, 2) is $(0.01)(1) + (0.99)(0.001)(2000) = 1.99$. Hence, the loop reduces the risk. In the limit, if the link (1,1) is traversed until an accident occurs, the risk will equal the consequence of an accident on that link, or 1. However, indefinite looping guarantees that the shipment will never reach the destination, which appears nonsensical.

Boffey and Karkazis (1995) point out that looping may reduce transport risk if one uses an exact model such as (2) or (3) to quantify risk. To avoid this, one could restrict the feasible set to loopless paths (as in Sivakumar, 1993a, and [Boffey and Karkazis, 1995](#)). This complicates the route selection problem considerably compared to the tractable shortest path formulation that results if one makes the simplifying assumption that results in model (1).

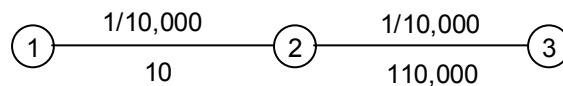


Figure 2: Example demonstrating that model (2) violates Axiom 2.

Figure 2 illustrates that model (2) violates Axiom 2. Here, the risk of the path $\{(1, 2), (2, 3)\}$ is $(0.0001)(10) + (0.9999)(0.0001)(110000) = 10.9999$. Any increase in the accident probability on link (1, 2) will *decrease* the risk of the path. If we sabotage the link (1, 2) and thus set its accident

probability to 1, we protect the link with the higher population, and the path risk is reduced to $(1)(10) = 10$. Hence, the “risk minimization” model promotes an accident.

The common thread in the examples in Figures 1 and 2 is the sacrifice of a low population link to protect a high population link. The model behaves in this manner because we allow it to “transfer credit” between links through memory of the probability of being able to reach the beginning of a link without accident. An increased probability of accident early on a path reduces the probability of an accident later. The $(1 - p_i)$ terms in (2) carry this “credit” forward along the path. If a link early in the path has a comparatively small population and experiences an increase in accident probability, then this will decrease the risk on later links and reduce the overall path risk. While model (2) represents reality more faithfully by accounting for the possibility that a shipment will never reach a particular link because of an accident earlier in the path, the model has some puzzling properties as evidenced by its violation of the axioms. In contrast, the exact version of the incident probability model satisfies the axioms since it does not involve link consequences (and derives no benefit from “transfer credit”). Table 2 provides a “violations summary” of the models in Table 1.

	Exact version violates axioms?	Approximate version violates axioms?
Traditional risk	Yes	No
Population exposure	No	N/A
Incident probability	No	No
Perceived risk	Yes	No
Mean – Variance	Yes	No
Disutility	Yes	No
Minimax	No	N/A
Conditional risk	Yes	Yes

Table 2: A summary of the relation between the eight risk expressions and the two axioms.

New Models

The axioms we presented and our investigation of whether different models satisfy them revealed the paradox that a model whose purpose is to select a minimum risk transport route may prescribe an action that will lead to an accident with certainty. Surprisingly, models that appear to describe

the real situation more faithfully have this paradoxical property while models that make a simplifying assumption (as in expression (1)) do not. In this section, we attempt to resolve this paradox. In doing so, we present a family of new expressions for quantifying hazmat transport risk.

The traditional risk model quantifies the risk resulting from a single truck traveling along a particular path. The truck either arrives safely at its destination or it has an accident. The model is myopic in the sense that it does not consider what happens after an accident occurs. In most instances the accident does not end the process. For example, if a truck carrying chlorine from a producer to a chemical processing plant has an accident that results in a spill of the cargo, the demand at the plant remains unsatisfied, and a second shipment must be made from the producer to the demand point. Furthermore, if the spilled substance can be recovered, it must be shipped to a hazardous materials treatment site, which results in one or more shipments (depending on how much ground material has to be scraped). Hence, an accident can never improve the system risk, and a model that encourages an accident is not an accurate reflection of reality.

In some cases, an accident will terminate the transport of the material. For example, if a truck carrying PCBs to a disposal site is involved in an accident resulting in a fire, the PCBs will be incinerated in the fire. However, when a hazardous material is being transported to fulfill a demand, an accident will result in the need to send another shipment to fulfill the demand. Hence, we must consider the possibility of multiple trips being needed to fulfill demand. The incident probability $IP(r)$ is intended to measure the probability that a trip along path r results in a release accident. The exact version of the incident probability path evaluation function is

$IP''(r) = 1 - \prod_i^n (1 - q_i)$ (using the notation $q_i = 1 - \exp(-p_i)$ defined earlier) and it measures the probability that transport along the path results in at least one accident. If this probability is independent of any previous trips that were terminated by an accident, then one can consider each trip as an independent trial with probability of “success” (i.e., the truck reaching the destination) equal to $1 - IP''(r)$. The number of trips required will then follow a geometric distribution with expected value $1/(1 - IP''(r))$. Since we are assuming that an accident terminates a trip, the expected consequence per trip will be $TR''(r)$ and the expected total consequence from all the trips required to fulfill demand will be

$$\text{TR}'''(r) = \frac{\text{TR}''(r)}{1 - \text{IP}''(r)} = \frac{\sum_{i=1}^n \prod_{j=1}^{i-1} (1 - q_j) q_i C_i}{\prod_{j=1}^n (1 - q_j)} = \sum_{i=1}^n q_i C_i \prod_{j=i}^n (1 - q_j)^{-1} \quad (4)$$

The last part of expression (4) has the following intuitive interpretation: the term $q_i C_i$ is the expected risk associated with traversing link i once and the term $\prod_{j=i}^n (1 - q_j)^{-1}$ is the expected number of times that link i (and subsequent links on the path) must be traversed before the shipment reaches the destination.

We now demonstrate that this model satisfies both axioms. We begin by showing that deleting the first or the last link in a path does not increase risk. If the first link is deleted, the risk is reduced by $q_1 C_1 \prod_{j=1}^n (1 - q_j)^{-1} \geq 0$. If the last link is deleted, the risk is reduced by:

$$\sum_{i=1}^n q_i C_i \prod_{j=i}^n (1 - q_j)^{-1} - \sum_{i=1}^{n-1} q_i C_i \prod_{j=i}^{n-1} (1 - q_j)^{-1} = q_n C_n (1 - q_n)^{-1} + q_n \sum_{i=1}^{n-1} q_i C_i \prod_{j=i}^n (1 - q_j)^{-1} \geq 0$$

Any subpath of r can be created by deleting one or more links from the beginning and/or the end of the path. Since deleting a link from the beginning or end of any path does not increase risk, any subpath of path r will have risk that is no greater than the risk of r and therefore expression (4) satisfies Axiom 1.

The derivatives of expression (4) with respect to the link attributes are:

$$\begin{aligned} \frac{\partial \text{TR}'''(r)}{\partial C_k} &= q_k \prod_{j=k}^n (1 - q_j)^{-1} \geq 0 \\ \frac{\partial \text{TR}'''(r)}{\partial p_k} &= \sum_{i=1}^{k-1} \frac{\partial}{\partial p_k} \left(q_i C_i \prod_{j=i}^n (1 - q_j)^{-1} \right) + \frac{\partial}{\partial p_k} \left(q_k C_k \prod_{j=k}^n (1 - q_j)^{-1} \right) \\ &= \sum_{i=1}^{k-1} q_i C_i \prod_{j=i, j \neq k}^n (1 - q_j)^{-1} \frac{\partial}{\partial p_k} \left((1 - q_k)^{-1} \right) + C_k \prod_{j=k+1}^n (1 - q_j)^{-1} \frac{\partial}{\partial p_k} \left(q_k (1 - q_k)^{-1} \right) \\ &= \sum_{i=1}^{k-1} q_i C_i \prod_{j=i}^n (1 - q_j)^{-1} + C_k \prod_{j=k}^n (1 - q_j)^{-1} \geq 0 \end{aligned}$$

Since these partial derivatives are non-negative, path risk is non-decreasing in link accident probabilities and consequences, which demonstrates that expression (4) satisfies Axiom 2.

Before discussing how the expression (4) can be optimized, we discuss an interesting property of the new model: it violates an optimality principle that simplifies many path selection problems.

Path selection optimality principle: Let $R(s,t)$ be the set of all paths from node s to node t . Suppose path r satisfies $v(r) = \min\{v(q) : q \in R(s,t)\}$. Consider two distinct nodes s' and t' on path r , where t' comes after s' , and let r' be the subpath of r from s' to t' . Then $v(r') = \min\{v(q) : q \in R(s',t')\}$.

This principle requires that all subpaths of an optimal path should themselves be optimal. It is a restatement of Bellman’s optimality principle for path selection models. All additive path evaluation functions (such as distance or time) satisfy this principle. [Erkut and Verter \(1998\)](#) suggest that all hazmat risk models should satisfy this principle (which they term Axiom 2). In Figure 3 we provide an example that demonstrates the violation of this principle by the new model.

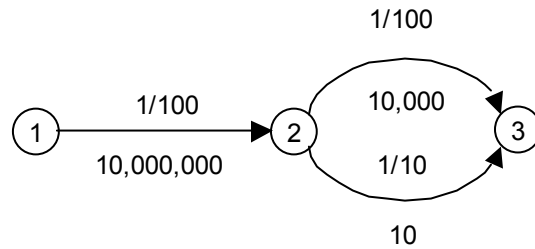


Figure 3: Example demonstrating that model (4) violates the path selection optimality principle. The optimal path from 2 to 3 is the bottom link while the optimal path from 1 to 3 takes the top link.

The violation demonstrated in Figure 3 makes intuitive sense. If the truck departs from node 2, then we compare the risks of the two paths between 2 and 3, and we take the path that is associated with lower risk. However, if the truck departs from node 1, then the problem is quite different. The risk associated with the (1,2) link dominates all other risks in the example. If the truck arrives without a release accident at node 2, but has an accident in the remaining portion of the path, then the second truck will have to traverse the link (1,2) again and impose the associated risks on the large

population. Hence, after arriving safely at node 2, it is optimal to take the minimum accident probability path the rest of the way (which minimizes the probability of the second trip). Given that link (1,2) is associated with the largest risk, at node 2 the truck acts in a way to protect the population along the link that has been completed. While this axiom may be reasonable for single hazmat trips, it cannot be expected to hold for models that allow multiple trips.

Model (4) is less tractable than the approximate traditional risk model (1). When compared to (2) and (3), (4) has the distinct advantage that it does not violate the two axioms. At first sight, (4) may appear to be equally difficult to optimize as (2) and (3). Furthermore, its violation of the path selection optimality principle rules out simple labeling algorithms. However, the problem of finding a path that minimizes (4) falls into a well-studied class of stochastic shortest path problems (Bertsekas, 2001). Figure 4 illustrates this problem, using the example network from Figure 3. This problem is an infinite horizon stochastic dynamic program and consequently solution approaches such as successive approximation or policy iteration can be applied to the minimization of (4) (see Bertsekas, 2001).

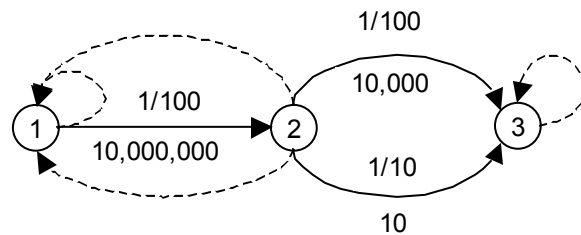


Figure 4: In the stochastic shortest path problem, each node in the transport network is viewed as a state in a Markov chain. The choice of a path determines the possible state transitions, which consist of successful traversals of links in the network, as well as transitions back to the origin node when an accident occurs (shown as dotted links). The choice of transport link from node 2 to node 3 determines which of the state transitions out of node 2 are included. The destination node is an absorbing state.

Concluding Remarks

The expressions for the perceived risk, mean-variance, and disutility models can all be modified in the same way as for the traditional risk model. All that is required is to divide the exact versions of the expressions shown in Table 1 with $1 - IP^r(r)$. The proof that (4) satisfies Axioms 1 and 2 applies to each of these models as well, if one redefines the link consequences appropriately. For example, for the mean-variance model, one would view $C_i(1 + kC_i^2)$ as the consequence for link i . The policy iteration algorithm can be applied to minimize these expressions as well.

Model (4) and similarly modified versions of some of the other expressions in Table 1 assume that shipments are sent from a given origin to a given destination until one shipment reaches the destination, and we have argued that this is appropriate when the purpose of the shipment(s) is to satisfy demand at the destination. In contrast, when the purpose of the transport is to transport hazardous waste to a disposal site, then it may not be appropriate to consider transport risk from a given origin to a given destination in isolation from the risk associated with storing the material at the origin, or disposing of the material at an alternate disposal site. The integrated modeling of these considerations is a topic worthy of further research.

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