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Standard Guide for
Conducting a Sensitivity Analysis for a Ground-Water Flow Model Application[1]

This standard is issued under the fixed designation D 5611; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This guide covers techniques that should be used to conduct a sensitivity analysis for a ground-water flow model. The sensitivity analysis results in quantitative relationships between model results and the input hydraulic properties or boundary conditions of the aquifers.

1.2 After a ground-water flow model has been calibrated, a sensitivity analysis may be performed. Examination of the sensitivity of calibration residuals and model conclusions to model inputs is a method for assessing the adequacy of the model with respect to its intended function.

1.3 After a model has been calibrated, a modeler may vary the value of some aspect of the conditions applying solely to the prediction simulations in order to satisfy some design criteria. For example, the number and locations of proposed pumping wells may be varied in order to minimize the required discharge. Insofar as these aspects are controllable, variation of these parameters is part of an optimization procedure, and, for the purposes of this guide, would not be considered to be a sensitivity analysis. On the other hand, estimates of future conditions that are not controllable, such as the recharge during a postulated drought of unknown duration and severity, would be considered as candidates for a sensitivity analysis.

1.4 This guide presents the simplest acceptable techniques for conducting a sensitivity analysis. Other techniques have been developed by researchers and could be used in lieu of the techniques in this guide.

1.5 This guide is written for performing sensitivity analyses for ground-water flow models. However, these techniques could be applied to other types of ground-water related models, such as analytical models, multi-phase flow models, non-continuum (karst or fracture flow) models, or mass transport models.

1.6 This guide is one of a series on ground-water modeling codes (software) and their applications, such as Guide D 5447 and Guide D 5490. Other standards have been prepared on environmental modeling, such as Practice E 978.

1.7 The values stated in inch-pound units are to be regarded as the standard. The SI units given in parentheses are for information only.

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

1.9 *This guide offers an organized collection of information or a series of options and does not recommend a specific course of action. This document cannot replace education or experience and should be used in conjunction with professional judgment. Not all aspects of this guide may be applicable in all circumstances. This ASTM standard is not intended to represent or replace the standard of care by which the adequacy of a given professional service must be judged, nor should this document be applied without consideration of a project's many unique aspects. The word "Standard" in the title of this document means only that the document has been approved through the ASTM consensus process.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rock, and Contained Fluids[2]

D 5447 Guide for Application of a Ground-Water Flow Model to a Site-Specific Problem[2]

D 5490 Guide for Comparing Ground-Water Flow Model Simulations to Site-Specific Information[2]

E 978 Practice for Evaluating Mathematical Models for the Environmental Fate of Chemicals[3]

3. Terminology

3.1 *Definitions:*

3.1.1 *boundary condition*—a mathematical expression of a state of the physical system that constrains the equations of the mathematical model.

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[1] This guide is under the jurisdiction of ASTM Committee D18 on Soil and Rockand is the direct responsibility of Subcommittee D18.21 on Ground Water and Vadose Zone Investigations.

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[2] *Annual Book of ASTM Standards*, Vol 04.08.

[3] *Annual Book of ASTM Standards*, Vol 11.05.

3.1.2 *calibration*—the process of refining the model representation of the hydrogeologic framework, hydraulic properties, and boundary conditions to achieve a desired degree of correspondence between the model simulations and observations of the ground-water flow system.

3.1.2.1 *Discussion*—During calibration, a modeler may vary the value of a model input to determine the value which produces the best degree of correspondence between the simulation and the physical hydrogeologic system. This process is sometimes called sensitivity analysis but for the purposes of this guide, sensitivity analysis begins only after calibration is complete.

3.1.3 *calibration targets*—measured, observed, calculated, or estimated hydraulic heads or ground-water flow rates that a model must reproduce, at least approximately, to be considered calibrated.

3.1.4 *ground-water flow model*—an application of a mathematical model to represent a ground-water flow system.

3.1.4.1 *Discussion*—This term refers specifically to modeling of ground-water hydraulics, and not to contaminant transport or other ground-water processes.

3.1.5 *hydraulic properties*—intensive properties of soil and rock that govern the transmission (that is, hydraulic conductivity, transmissivity, and leakance) and storage (that is, specific storage, storativity, and specific yield) of water.

3.1.6 *residual*—the difference between the computed and observed values of a variable at a specific time and location.

3.1.7 *sensitivity*—the variation in the value of one or more output variables (such as hydraulic heads) or quantities calculated from the output variables (such as ground-water flow rates) due to variability or uncertainty in one or more inputs to a ground-water flow model (such as hydraulic properties or boundary conditions).

3.1.8 *sensitivity analysis*—a quantitative evaluation of the impact of variability or uncertainty in model inputs on the degree of calibration of a model and on its results or conclusions.[4]

3.1.8.1 *Discussion*—Anderson and Woessner[4] use "calibration sensitivity analysis" for assessing the effect of uncertainty on the calibrated model and ′′prediction sensitivity analysis" for assessing the effect of uncertainty on the prediction. The definition of sensitivity analysis for the purposes of this guide combines these concepts, because only by simultaneously evaluating the effects on the model's calibration and predictions can any particular level of sensitivity be considered significant or insignificant.

3.1.9 *simulation*—one complete execution of a ground-water modeling computer program, including input and output.

3.2 For definitions of other terms used in this guide, see Terminology D 653.

4. Significance and Use

4.1 After a model has been calibrated and used to draw conclusions about a physical hydrogeologic system (for example, estimating the capture zone of a proposed extraction well), a sensitivity analysis can be performed to identify which model inputs have the most impact on the degree of calibration and on the conclusions of the modeling analysis.

4.2 If variations in some model inputs result in insignificant changes in the degree of calibration but cause significantly different conclusions, then the mere fact of having used a calibrated model does not mean that the conclusions of the modeling study are valid.

4.3 This guide is not meant to be an inflexible description of techniques of performing a sensitivity analysis; other techniques may be applied as appropriate and, after due consideration, some of the techniques herein may be omitted, altered, or enhanced.

5. Sensitivity Analysis

5.1 The first step for performing a sensitivity analysis is to identify which model inputs should be varied. Then, for each input: execute calibration and prediction simulations with the value of the input varied over a specified range; graph calibration residuals and model predictions as functions of the value of the input; and determine the type of sensitivity that the model has with respect to the input.

5.2 *Identification of Inputs to be Varied*:

5.2.1 Identify model inputs that are likely to affect computed hydraulic heads and ground-water flow rates at the times and locations where similar measured quantities exist, and thereby affect calibration residuals. Also, identify model inputs that are likely to affect the computed hydraulic heads upon which the model's conclusions are based in the predictive simulations.

5.2.2 Usually, changing the value of an input at a single node or element of a model will not significantly affect any results. Therefore, it is important to assemble model inputs into meaningful groups for variation. For example, consider an unconfined aquifer that discharges into a river. If the river is represented in a finite-difference model by 14 nodes, then varying the conductance of the river-bottom sediments in only one of the nodes will not significantly affect computed flow into the river or computed hydraulic heads. Unless there are compelling reasons otherwise, the conductance in all river nodes should be varied as a unit.

5.2.3 Coordinated changes in model inputs are changes made to more than one type of input at a time. In ground-water flow models, some coordinated changes in input values (for example, hydraulic conductivity and recharge) can have little effect on calibration but large effects on prediction. If the model was not calibrated to multiple hydrologic conditions, sensitivity analysis of coordinated changes can identify potential non-uniqueness of the calibrated input data sets.

5.3 *Execution of Simulations*:

5.3.1 For each input (or group of inputs) to be varied, decide upon the range over which to vary the values. Some input values should be varied geometrically while others should be varied arithmetically. The type of variation for each input and the range over which it is varied are based on the modeler's judgment, with the goal of finding a Type IV sensitivity (see 5.5.1.4) if it exists.

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[4] Anderson, Mary P., and Woessner, William W., *Applied Groundwater Modeling—Simulation of Flow and Advective Transport*, Academic Press, Inc., San Diego, 1992.

NOTE 1—If the value of a model input (or group of inputs) was measured in the field, then that input need only be varied with the range of the error of the measurement.

5.3.2 For each value of each group of inputs, rerun the calibration and prediction runs of the model with the new value in place of the calibrated value. Calculate the calibration residuals (or residual statistics, or both) that result as a consequence of using the new value. Determine the effect of the new value on the model's conclusions based on using the new value in the prediction simulations.

5.4 *Graphing Results*:

5.4.1 For each input (or group of inputs), prepare a graph of the effect of variation of that parameter upon calibration residuals and the model's conclusions. Figs. 1-4 show sample graphs of the results of sensitivity analyses.

5.4.2 Rather than display the effect on every residual, it may be more appropriate to display the effect on residual statistics such as maximum residual, minimum residual, residual mean, and standard deviation of residuals (see Guide D 5490).

5.4.3 In some cases, it may be more illustrative to present contours of head change as a result of variation of input values. In transient simulations, graphs of head change versus time may be presented.

5.4.4 Other types of graphs not mentioned here may be more appropriate in some circumstances.

5.5 *Determination of the Type of Sensitivity*:

5.5.1 For each input (or group of inputs), determine the type of sensitivity of the model to that input. There are four types of sensitivity, Types I through IV, depending on whether the changes to the calibration residuals and model's conclusions are significant or insignificant. The four types of sensitivity are described in the following sections and summarized on Fig. 5.

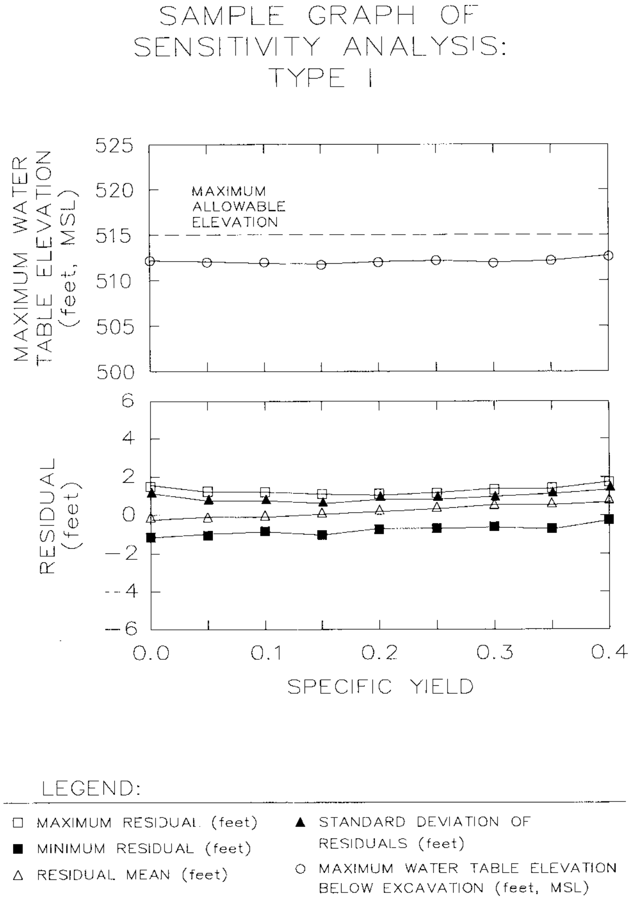

FIG. 1 Sample Graph of Sensitivity Analysis, Type I Sensitivity

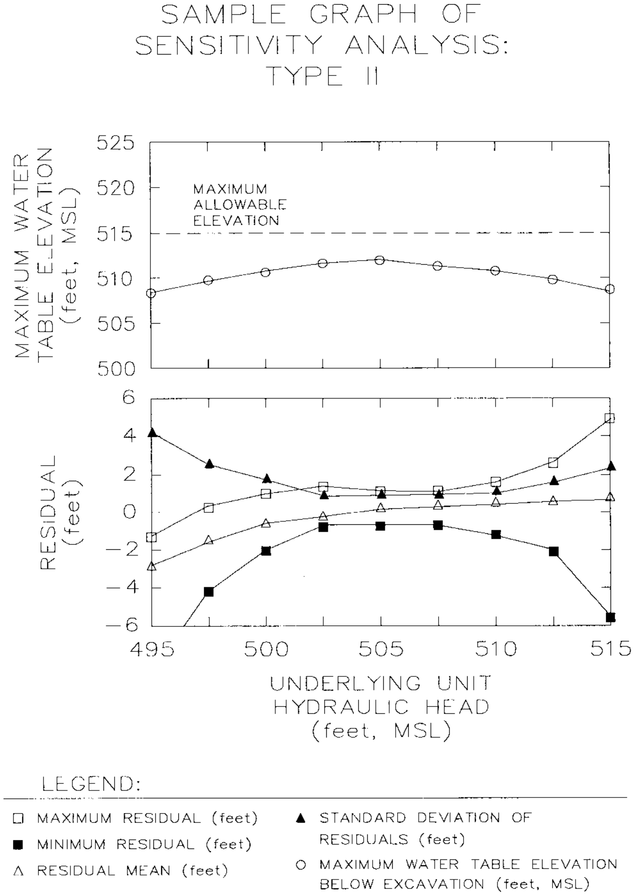

FIG. 2 Sample Graph of Sensitivity Analysis, Type II Sensitivity

NOTE 2—Whether a given change in the calibration residuals or residual statistics is considered significant or insignificant is a matter of judgment. On the other hand, changes in the model's conclusions are usually able to be characterized objectively. For example, if a model is used to design an excavation dewatering system, then the computed water table is either below or above the bottom of the proposed excavation.

5.5.1.1 *Type I Sensitivity*—When variation of an input causes insignificant changes in the calibration residuals as well as the model's conclusions, then that model has a Type I sensitivity to the input. Fig. 1 shows an example of Type I sensitivity. Type I sensitivity is of no concern because regardless of the value of the input, the conclusion will remain the same.

5.5.1.2 *Type II Sensitivity*—When variation of an input causes significant changes in the calibration residuals but insignificant changes in the model's conclusions, then that model has a Type II sensitivity to the input. Fig. 2 shows an example of Type II sensitivity. Type II sensitivity is of no concern because regardless of the value of the input, the conclusion will remain the same.

5.5.1.3 *Type III Sensitivity*—When variation of an input causes significant changes to both the calibration residuals and the model's conclusions, then that model has a Type III

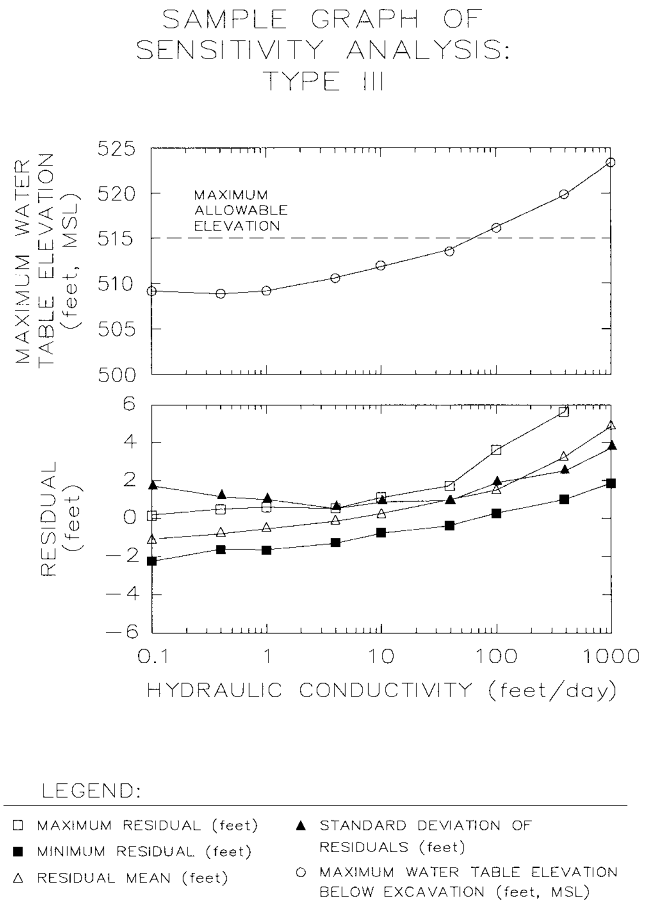

FIG. 3 Sample Graph of Sensitivity Analysis, Type III Sensitivity

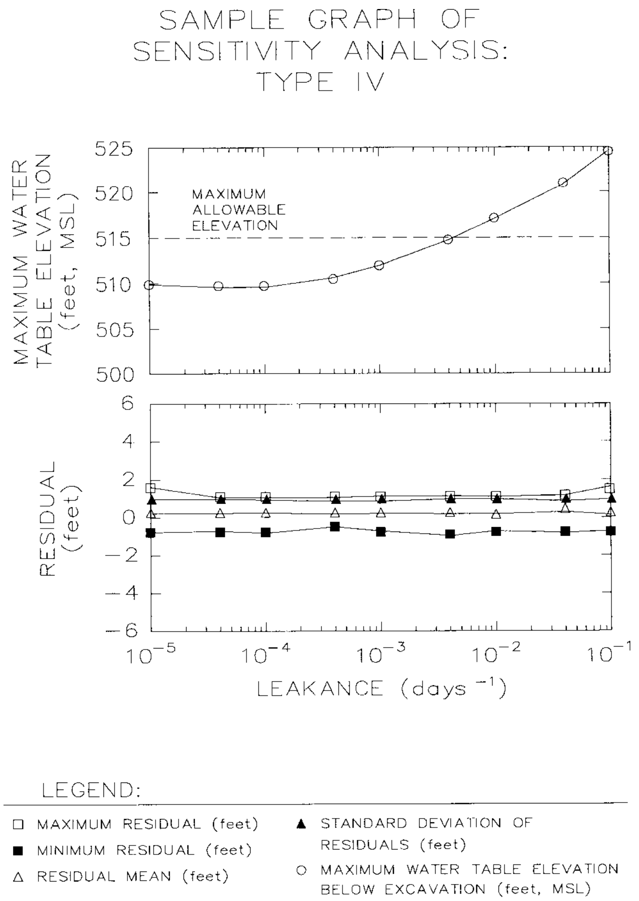

FIG. 4 Sample Graph of Sensitivity Analysis, Type IV Sensitivity

sensitivity to the input. Fig. 3 shows an example of Type III sensitivity. Type III sensitivity is of no concern because, even though the model's conclusions change as a result of variation of the input, the parameters used in those simulations cause the model to become uncalibrated. Therefore, the calibration process eliminates those values from being considered to be realistic.

5.5.1.4 *Type IV Sensitivity*—If, for some value of the input that is being varied, the model's conclusions are changed but the change in calibration residuals is insignificant, then the model has a Type IV sensitivity to that input. Fig. 4 shows an example of Type IV sensitivity. Type IV sensitivity can invalidate model results because over the range of that parameter in which the model can be considered calibrated, the conclusions of the model change. A Type IV sensitivity generally requires additional data collection to decrease the range of possible values of the parameter.

5.5.2 Some input parameters (for example, the hydraulic conductivity of a proposed cutoff wall) are used only in the prediction simulations. In such a case, the sensitivity is automatically either Type III or IV, depending on the significance of the changes in the model's conclusions. If Type IV, supporting documentation for the value of the parameter used in the prediction simulations is necessary (but not necessarily sufficient) to justify the conclusions of the model.

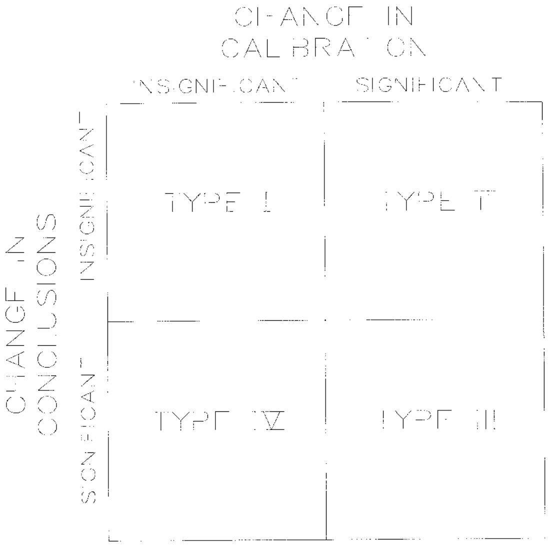

FIG. 5 Summary of Sensitivity Types

6. Report

6.1 If a sensitivity analysis is not performed, the report should state why a sensitivity analysis was not needed. If a sensitivity analysis is performed, the report should state which model inputs were varied and which computed outputs were examined. The report should justify the selection of model inputs and computed outputs in terms of the modeling objective.

6.2 For each model input that was varied, the report should present a graph showing the changes in residuals (or residual statistics) and the computed outputs with respect to changes in the model input. The report should either state that none of the analyses had a Type IV result, or else identify which analyses had Type IV results.

7. Keywords

7.1 calibration; computer; ground water; modeling; sensitivity

APPENDIX

(Nonmandatory Information)

X1. EXAMPLE SENSITIVITY GRAPHS

X1.1 Consider a hypothetical ground-water flow model used to design an excavation dewatering system. The bottom of the excavation will be at an elevation of 520 ft (158.5 m) above mean sea level (MSL), and the water table must be at least 5 feet below the excavation floor, or no more than 515 ft (157.0 m) MSL. Four parameters are selected for sensitivity analysis: the specific yield of a sand unit, hydraulic conductivity of the sand unit, the leakance of a clay unit, and the hydraulic head in an underlying silty sand unit. Figs. 1-4 show sample graphs of the results of sensitivity analyses performed on these parameters.

X1.1.1 Fig. 1 shows the results of a sensitivity analysis performed on the specific yield of the sand unit. The calibrated value was 0.2. As the specific yield was varied from 0.0 to 0.4, neither the calibration residuals nor the model conclusion varied significantly as a result of variation in the specific yield. Therefore the model has Type I sensitivity to specific yield.

X1.1.2 Fig. 2 shows the results of a sensitivity analysis performed on the hydraulic head of an underlying unit. The calibrated value was 505 ft (153.9 m) MSL. As the hydraulic head was varied from 495 to 515 ft (150.9 to 157.0 m), MSL, the residuals statistics degraded significantly. However, although the maximum water table elevation below the excavation changed, the conclusion of the model (that the excavation would stay dry) did not change. Therefore the model has Type II sensitivity to the hydraulic head in the underlying unit.

X1.1.3 Fig. 3 shows the results of a sensitivity analysis performed on the hydraulic conductivity of the sand unit. The calibrated value of the hydraulic conductivity was 10 ft (3.05 m/d) per day and it was varied from 0.1 to 1000 ft (0.03 to 304.8 m/d) per day. As the hydraulic conductivity exceeded 50 feet per day, the water table below the excavation increased to above 515 ft (157.0 m), MSL. However, the calibration residuals also increased, so that the model could no longer be considered calibrated. Therefore, the fact that the model's conclusion changed (that is, for some values of the parameter, the excavation was no longer dry) is unimportant. This is an example of Type III sensitivity.

X1.1.4 Fig. 4 shows the results of a sensitivity analysis performed on the leakance of an underlying clay unit. The calibrated value was 10^{-3} days^{-1}. As the leakance was varied from 10^{-5} to 10^{-1} days^{-1}, the calibration residuals remained practically constant. However, at the higher leakances, the excavation was not dewatered. Therefore, the conclusion of the model varied significantly while the calibration did not. This is a Type IV sensitivity, and it invalidates the use of the model for design of the excavation dewatering system until the actual value of the leakance can be determined.

X1.2 Fig. 5shows a summary of the four types of sensitivity and the conditions under which they occur.